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GENERATING QUANTITATIVE DATA REQUIREMENTS
FOR PRICING "PUBLICLY PROVIDED" GOODS AND SERVICES

Katsuaki Terasawa and David Whipple

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Economists and public sector managers have long faced the problem of having to recommend social courses of action which are not "Pareto Superior" (i.e. make someone better off but at another's expense). In the absence of a well defined objective function covering the possibilities under consideration, value judgments, based only on qualitative information many times, were needed.

In the present paper we demonstrate that there exists a set of public policy decisions which, using the implications of previously determined political

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) decisions, demand only quantitative market information. These are decisions concerning the fee structure of government (municipal, state or federal) facilities which have characteristics of both private and public goods.

An example of the relatively straightforward nature of the necessary data to determine the optimal fee structure of a municipal swimming pool is constructed and used to illustrate the major points of the paper.

I. INTRODUCTION.

Social welfare functions have long been the subjects of, and stumbling blocks for, research into the optimal pattern of governmental intervention in the supposedly unfettered market economy. If there existed a mapping from cardinal individual welfare space onto the real line, the choice of optimal social policies would be much simplified. However the non-existence of such a construct and the substantiating reasons are now well known.

But the government does intercede in the private sector in myriad ways. A major justification for these actions, if not for their specific form and magnitude, is the existence of public goods or externalities--i.e., those particular goods and services which enter into the utility calculations of many more than the specific individuals who directly consume the good or service, and the inability to restrict the (dis)satisfaction to those consumers--the "free rider" problem.

In addition, there exists a large group of publicly run facilities which fit Samuelson's definition of those for which it is technically possible to limit consumption to the specified group of individuals. Thus one might be tempted to "...convert [the] public good into a private good, and ... side-step the vexing problem of collective expenditure, instead relying on the free pricing mechanism."[1] For the purposes of this paper we will denote these as "publicly provided" goods. Elements of this set include zoos, municipal swimming pools, libraries, museums, etc., and are in general characterized by negligible uniform (with respect to time) user fees. These charges are usually set by decentralized decision makers appointed or elected to oversee

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[1] "Aspects of Public Expenditure Theories," Review of Economics and Statistics, November 1958, p. 335.

the operation of the specific facility or related class of facilities. They take as given the size of existing facilities and the approved budget passed on by a more centralized decision group and attempt to live within these financial bounds by setting user fees accordingly.[2] This involves optimization according to an objective function, whether explicitly or implicitly. It appears to the authors that many times the criterion function used is administrative simplicity (which is certainly not necessarily synonymous with minimizing administrative costs or maximizing net revenue). Thus, because of custom or lack of the disposition or expertise to analyze the full price/use/revenue possibilities, uniform fee structures are adopted.

We will attempt to show in this paper that for many of the goods which are publicly provided, a differentiated fee structure over time may well be pareto superior to one which is uniform. That is, by effectively redistributing income through the fee structure when lump-sum transfers are beyond the realm of feasibility, social welfare may be enhanced. This is accomplished by varying the user fee by time period when it is not possible to do so by income class directly.

To summarize, we will argue that, given that the subset of goods under discussion are provided by the government (at whatever level) when the market either also provides them or could have provided them (although in different quantities), the reason for such intervention becomes important for pricing

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[2] We do not address the optimal size of the facility problem in that we are primarily interested in the entry fee determination problem for existing public facilities faced by the specific governmental group charged with its operation.

This and other restrictions on the scope of the present paper are adopted in an attempt to constructively react to criticisms of the literature such as that of Houthakker (1958, p. 464).

> "Much of the sterility of modern welfare economics can be attributed to the neglect of institutional and technological constraints which in reality make the range of possible solutions much narrower than the theories would have us believe."

purposes. Many times by their very nature certain of these goods are intended mainly for the consumption of a particular segment of the population. For example, if there exist alternative private substitutes then the good is publicly provided in order to directly subsidize its consumption by a particular group. This may be a locale or income class. When this is the case (as say in building a community swimming pool in a particular neighborhood) then the original reason for the existence of the facility may be obscured by an inappropriate fee structure which causes its use by members of the target group to fall short of that envisioned when its size was determined.[3]

In other cases, the facility is provided not specifically for one group, but because of the nature of the particular good or service itself (e.g., a museum or public library). In this case we will examine the conditions under which, if a fee is charged, it is optimal to differentiate according to time period.

The general nonoptimality of a uniform fee structure in peak load type situations is well known. In practice however, this has only been applied to goods and services produced under private auspices (even if publicly regulated). In particular, with publicly provided goods such as those mentioned earlier, uniformity is certainly the rule. To the extent this is based on recognition of significant administrative costs it is totally reasonable. However, in most cases it appears to be a matter of ignorance on the part of the decision makers of some basic economic principles.

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[3]Thus we are considering the optimal pricing of goods which Samuelson has identified as "paternalistic" in that they "are voted upon themselves by a democratic people because they do not regard the results from spontaneous market action as optimal." [Samuelson, 1955.]

That which we propose in this paper has much in common with the philosophy embodied in Feldstein's recent paper.[4] There he reiterates that in the two part tariff literature, the fixed, uniform portion of the tariff has been criticized as an essentially regressive head tax and notes that "what is needed is a pricing rule that balances efficiency and distributional equity subject to the constraint that every consumer must pay the same marginal price and fixed charge."[5]

We too are concerned with the welfare implications of the uniform fee structure, in this case of publicly provided goods and services. However, we have chosen to examine the potential welfare gains available to a "community" (however defined) which allows the fee structure of the public facility to be determined by differential periodic demands, while recognizing that total fees collected must satisfy exogeneous revenue constraints.

Finally, we conclude by indicating that, in cases which fit the criterion we set forth, it is possible to rely upon relatively readily available quantitative market data in setting pricing policies which are socially more desirable than that presently in use.

In the following section, we construct a model and examine the points just raised. We have chosen to use the management of a municipal swimming pool as the vehicle for discussion of the various aspects of the analytical structure in terms of an existing and intuitively obvious problem. The model is easily applicable to various other sorts of public pricing areas however.

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[4] "Equity and Efficiency in Public Sector Pricing: The Optimal Two Part Tariff," QJE May 1972.

[5] Ibid., p. 175.

II. A MODEL.

Consider a decision maker (DM) who is charged with determining the fee structure covering individual use of a public swimming pool. Assume the facility was built with the expressed intent of helping a certain segment of the population. For example, suppose we are dealing with a community which contains both private, for-profit swim clubs and family-owned swimming pools. The decision to build a municipal swimming pool is usually motivated by the fact that a particular (in this case, income) group does not have effective economic access to swimming facilities. In most cases, partial federal funding is available for such projects under the condition that the facility is geographically accessible to low income groups.

However, the DM cannot for various social and legal reasons "antagonize" the non-target groups in the population. This implies access must be controlled in an objective manner. This in turn implies that the only rationing mechanism will be price, ruling out such schemes such as "membership" cards issued only to neighborhood residents, low income families, etc.

Assume there exist K groups in the society and J swimming periods. We further assume different fees can be collected on the basis of age differentials and swimming periods, but not on the basis of class differentials. The constraints the DM faces are:

(i) a revenue constraint, i.e., the revenue from fees collected must cover some specified portion of the costs of operating the facility. This proportion γ, is exogeneously given to the DM in the departmental budget approved by, in this case, the city council.

(ii) a utility constraint, in that we do not wish to construct a fee structure which would make any of the non-target groups any worse off than they were before the pool began operation.[6]

Then we define:

P_{ij} as the fee charged persons in the i^{th} age group to swim in the j^{th} period. For example we may realistically have three age groups, children, adults, and senior citizens, say. The J periods cover, at least, the peak demand and off-peak demand swimming periods each day of the planning period. Thus if we had morning, afternoon and evening swimming sessions every day and the planning period was the 100 day "summer," then $j = 1,\ldots,300$.

$P = (P_{11},\ldots,P_{ij},\ldots,P_{IJ})$

$x^k(P)$ as the demand for the Hicksian good by the k^{th} social group.

$s^k_{ij}(P)$ as the aggregate demand for swimming in the j^{th} period by the i^{th} age group in the k^{th} social group. Thus s^1_{1j} may be the demand of low income children as a group for swimming in the j^{th} period, while s^2_{3j} might be the total amount of swimming demanded by affluent senior citizens in the j^{th} period, etc.

S as the capacity of the swimming area in terms of square feet of pool surface.

δ^i as the unit swimming space requirement for members of the i^{th} age group.

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[6] Clearly here we are dealing with only the effect of the fee structure on the non-target groups, recognizing that their tax bill may have changed as a result of the facility having been built. This however, is outside the scope of the DM's problem. For a discussion of the financing of such a facility through non-user fees see Whipple [1974].

s_j as the "total demand" for swimming in period j, i.e., $s_j = \sum_i \sum_k \delta^i s^k_{ij}$

$s^k = (s^k_{11},\ldots,s^k_{ij},\ldots,s^k_{IJ})$

$s = (s_1,\ldots,s_J)$

$C(s,S)$ as the operating cost of providing swimming services in the pool which depends upon both its periodic use and the size of the facility, with $\frac{\partial C}{\partial s_j} \frac{\partial s_j}{\partial s_{ij}} \geq 0$ and $\frac{\partial C}{\partial s_j} \to \infty$ as $s_j \to S + \epsilon$, $\forall\ \epsilon$ arbitrarily small for any $j \in J$. This then says that the specified cost function includes a "capacity constraint" on use of the pool in each period.[7] This ensures that the periodic price must be set such that the capacity of the pool is not violated.

Without loss of generality, given the stated purposes of this paper, we assume that the K social groups contain homogeneous members and thus consider social group indirect utility functions $U^k[s^k(P), x^k(P)]$, $k \in K$ rather than those of individual members. Let h denote the target group.

The problem facing the DM is then to

$$\underset{\text{w.r.t } P}{\text{Maximize}} \quad U^h[s^h(P),x^h(P)]$$

$$\text{Subject to} \quad \text{(i)} \quad \sum_i \sum_j \{\sum_k s^k_{ij}(P)\} P_{ij} \geq \gamma C(s,S)$$

$$\text{(ii)} \quad U^k[s^k(P),X^k(P)] \geq \bar{U}^k, \qquad \forall k \neq h,\ k \in K$$

where $\bar{U}^k$ is some given positive number associated with that groups utility before the pool was opened.

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[7]We must note at this point that the present paper by design does not deal with the problem of congestion and the associated negative externalities in the use of the pool. The definition of δ and S as single deterministic numbers stems from the authors' desire to focus attention on the possibilities and reasons for differentiated fee structures which is a problem separable from the congestion question.

Then the first order conditions for an interior maximization take the form:

$$\frac{\partial U^h}{\partial s^h}\frac{\partial s^h}{\partial P_{mn}} + \frac{\partial U^h}{\partial X^h}\frac{\partial X^h}{\partial P_{mn}} - \sum_j \lambda^j \frac{\partial s_j}{\partial P_{mn}} + \eta\{\sum_i \sum_j P_{ij}(\sum_k \frac{\partial s^k_{ij}}{\partial P_{mn}}) + (\sum_k s^k_{mn}) \tag{1}$$

$$- \gamma \sum_j \frac{\partial C}{\partial s_j}\frac{\partial s_j}{\partial P_{mn}}\} + \sum_{k \neq h} \mu^k (\frac{\partial U^k}{\partial s^k}\frac{\partial s^k}{\partial P_{mn}} + \frac{\partial U^k}{\partial X^k}\frac{\partial X^k}{\partial P_{mn}}) = 0$$

$$\forall\ m \in I,\quad n \in J$$

where:

$$\frac{\partial U^k}{\partial s^k} = (\frac{\partial U^k}{\partial s^k_{11}}, \ldots, \frac{\partial U^k}{\partial s^k_{IJ}}) \qquad \forall\ k \in K$$

$$\frac{\partial s^k}{\partial P_{mn}} = (\frac{\partial s^k_{11}}{\partial P_{mn}}, \ldots, \frac{\partial s^k_{IJ}}{\partial P_{mn}}) \qquad \forall\ k \in K \tag{2}$$

$$\frac{\partial s_j}{\partial P_{mn}} = \sum_i \sum_k \delta^i \frac{\partial s^k_{ij}}{\partial P_{mn}} \qquad \forall\ j \in J$$

and η and μ^k are the Lagrange multipliers associated with the revenue and utility constraints respectively. Given our homogeneity assumptions regarding the social group members (1) may be written as

$$\sum_i \sum_j \sum_k \frac{\partial s^k_{ij}}{\partial P_{mn}}\{\eta P_{ij} - (\gamma \delta_i \eta \frac{\partial C}{\partial s_j})\} = \sum_k \{s^k_{mn}(\alpha^k \beta^k - \eta)\} \tag{3}$$

$$\forall\ m \in I,\ n \in J \qquad \text{where} \begin{cases} \beta^k = \mu^k & \forall\ k \neq h \\ \beta^h = 1 & \end{cases}$$

and where α^k is marginal utility of income for k^{th} budgetary unit.

We may restrict our discussion to those cases in which the revenue constraint is truly effective, i.e., in which $\eta > 0$, both because they are the most interesting and because, mathematically, if $\eta = 0$, (3) implies that $\beta^k = 0, \forall\ k \neq h$, which says that $\mu^k = 0, \forall\ k \neq h$. But this violates the assumption that swimming is viewed as a "good" by the target and hence cannot obtain.[8]

Since the revenue constraint is "truly effective," i.e. constraint (i) above holds with equality and the unconstrained optimum would have been different without (i)'s presence, η is strictly positive and, if the (IJ) by (IJ) gross substitute matrix $\left[\sum_k \frac{\partial s_{ij}^k}{\partial P_{mn}}\right] = A$ is non-singular, we can solve explicitly for the elements of the optimal fee structure.

$$P_{mn} = \sum_i \sum_j \sum_k s_{ij}^k \frac{\alpha^k \beta^k - \eta}{\eta} \frac{A_{ij,mn}}{|A|} + \delta^m \gamma \frac{\partial C}{\partial s_n} \tag{4}$$

(where the $A_{ij,mn}$ are the cofactors of A). Thus the first term in the right hand side of equation (4) reflects distributional or equity considerations and the second term cost and capacity or efficiency considerations, as will be discussed later.

[8] Proof of this statement is straightforward and is therefore omitted in the interest of brevity.

III. THE OPTIMAL FEE STRUCTURE

We now examine the possibility that the fee structure satisfying the reduced form optimality conditions (4) will exhibit uniformity, i.e., that $P_{\bar{i}j} = P_{\bar{i}n}$ $\forall$ $j,n \in J$ and $\forall$ $\bar{i} \in I$. That is we are defining uniformity with respect to the time period <u>within each</u> given age group. In particular, we will show that such a case, while not impossible, is unlikely to occur.

<u>Theorem 1</u>. The fee structure will be proportional to marginal cost if (i) the income distribution is optimal in the sense that the target groups' welfare change due to a change in the magnitude of the revenue constraint equals the weighted marginal utility of income of each of the other social groups.

To prove Theorem 1 (and to facilitate future discussion) we first prove the following Lemma.

<u>Lemma 1</u>. If optimal lump sum income transfers are possible such that statement (i) in theorem 1 holds then $\alpha^k \beta^k = \eta$.

<u>Proof</u>. Consider the two related maximization problems:

I. Maximize the utility of the k^{th} social group subject to the group budget constraint:

$$\max_{\text{wrt } s_{ij}^k, x^k} \quad U^k(s^k, x^k)$$

$$\text{Subject to} \quad \sum_i \sum_j P_{ij} s_{ij}^k + x^k = \bar{y}^k - t^k \tag{5}$$

where t^k is the lump sum tax (transfer). Let α^k be the multiplier associated with the constraint in the Lagrangian.

II. The original maximization problem set forth on page 7 above, but with a modified revenue constraint as below to account for the lump sum tax possibility explicitly:

$$\sum_i \sum_j P_{ij}\left(\sum_k s_{ij}^k(P,t^k)\right) + \sum_k t^k = \gamma C(s,S)$$

From the first order conditions for problem I we know $\frac{\partial U^k}{\partial s_{ij}^k} = \alpha^k P_{ij}$ and $\alpha^k = \frac{\partial U^k}{\partial x^k}$, and differentiating (5) with respect to t^k and P_{mn} respectively we get:

$$\sum_i \sum_j P_{ij} \frac{\partial s_{ij}^k}{\partial t^k} + \frac{\partial x^k}{\partial t^k} = -1 \tag{6}$$

$$\sum_i \sum_j P_{ij} \frac{\partial s_{ij}^k}{\partial P_{mn}} + s_{mn}^k + \frac{\partial x^k}{\partial P_{mn}} = 0 \tag{7}$$

The first order conditions for problem II include

$$- \sum_k \alpha^k \beta^k s_{mn}^k + \eta \left[\sum_i \sum_j \sum_k \left(\frac{\partial s_{ij}^k}{\partial P_{mn}}\right) + \sum_k s_{mn}^k - \gamma \sum_i \sum_j \sum_k \delta_i \frac{\partial s_{ij}^k}{\partial P_{mn}} \frac{\partial C}{\partial s_j} \right] = 0 \tag{8}$$

$$- \alpha^k \beta^k + \eta \left[\sum_i \sum_j P_{ij} \left(\frac{\partial s_{ij}^k}{\partial t^k}\right) + 1 - \gamma \sum_i \sum_j \frac{\partial C}{\partial s_j} \delta_i \frac{\partial s_{ij}^k}{\partial t^k} \right] = 0 \tag{9}$$

substituting α^k and recalling the definition of β^k as before. Again substituting from (6) and (7), manipulating (8) and (9) yields (10) below, given the following definitions:

$$b^k_{ij,mn} = \frac{\partial s^k_{ij}}{\partial P_{mn}} - \frac{\partial s^k_{ij}}{\partial t^k} s^k_{mn}$$

and

$$b_{ij,mn} = \sum_k b^k_{ij,mn}$$

$$[b_{ij,mn}](P_{ij} - \gamma\delta_i \frac{\partial C}{\partial s_j}) = (0) \tag{10}$$

Noting that $[b_{ij,mn}]$ is the net substitute matrix (in the present context) and hence non-singular implies

$$P_{ij} = \gamma\delta_i \frac{\partial C}{\partial s_j} \qquad \forall\ i, j$$

Substituting for P_{ij} in (9) collapses the bracketed term to 1 $\ni$ (9) appears:

$$\alpha^k\beta^k = \eta$$

Q.E.D.

The proof of Theorem 1 follows directly from Lemma 1.

<u>Theorem 2</u>. If the total amount of income allocated to the purchase of swimming remains constant with changes in the periodic swimming fees then these fees will be proportional to periodic marginal cost.[9]

<u>Proof</u>. Noting that the assumption implies $\frac{\partial x^k}{\partial Pmn} = 0\ \forall\ m, m, k$ in the neighborhood of the optimal values of the P_{ij}'s, and that using we may derive a slightly altered version of (4) as follows:

$$P_{mn} = \sum_i \sum_j \left[\left\{\sum \frac{\partial x^k}{\partial P_{mn}} \left(\eta - \alpha^k\beta^k\right)\right\} \frac{B_{ij,mn}}{|B|}\right] + \eta\gamma\delta_m \frac{\partial C}{\partial s_n} \tag{11}$$

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[9]This corresponds to Lerner's "first rule of ideal taxation" [5] in which he maintains that proportionality will be kept if the shifting of resources which results from the imposition of a tax is <u>within</u> the taxed sector.

where $B_{ij,mn}$ is the cofactor of the IJ x IJ gross substitution matrix B of the form $\sum_k \frac{\partial s_{ij}^k}{\partial P_{mn}} \alpha^k \beta^k$ and $|B|$ is the determinant of B. The result is immediate.[10]

The immediate impact of theorems 1 and 2 is that under fairly restrictive assumptions the determination of the optimal fee structure is based solely upon efficiency rather than equity considerations. Thus the present model by considering the group welfare instead of that of the individual as in Dixit [2], and by considering the distributional problems associated with η being different than $\alpha^k\beta^k$, which was not examined by Baumol and Bradford [1] who implicitly assumed $\eta = \alpha\beta = 1$, we are able to derive an expression for the effect of equity considerations on the determination of the optimal price or fee structure.

However, even in the cases above where equity considerations are made unnecessary in the pricing decision, there is no guarantee that uniformity is optimal, as the peak load pricing literature indicates. That is, with differential demands - the existence of peak and off-peak periods - uniform pricing is in general suboptimal.

Adding in the cases where the necessity for equity considerations exists then will tend to make a uniform optimal fee structure even more unlikely.

The question which remains to be answered concerns the sensitivity of this non-uniformity tendency to changes in the proportion of costs which must be covered by fee revenue.

Assume that all costs must be covered (i.e., γ=1), that a uniform fee structure is optimal with the given demand situations etc., and that the pool

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[10] We note that equation (11) may be viewed as a generalized version of the fundamental equation (11) in Dixit [2], or the "general rule," equation (9), in Baumol and Bradford [1].

will operate at capacity in at least one, but not all periods. Then, in the absence of the shifting peak problem, this existence of peak and off-peak periods guarantees that the optimal fee structure will never be uniform with $\gamma < 1$.

To see this recall that the periodic price is used to insure that the capacity constraint is not violated. Thus if γ is reduced, such that total required revenue falls, the price in the "maximal peak period" -- that at capacity already -- cannot be reduced to the same degree as that in the "off-peak" periods, which must be reduced as much as possible in order to maximize utility of the social groups. Thus if the original fee structure was uniform over the periods, it no longer will be, for any $\gamma < 1$.

To consider the general case of the sensitivity of the periodic prices to changes in the size of required revenue, (assuming linearity of the demand functions for simplicity) we differentiate (4) to derive the following expression for the result of such changes in γ:

$$\frac{\partial P_{mn}}{\partial \gamma} = \frac{\delta_m \dfrac{\partial C}{\partial s_j}}{1 - \sum\limits_{i,j} \left\{ \sum \dfrac{\partial s^k_{ij}}{\partial P_{mn}} \alpha^k \beta^k - \eta \right\} \dfrac{A_{ij,mn}}{|A|}} \tag{12}$$

$$- \delta_m \gamma \sum_t \delta \frac{\partial s_{tn}}{\partial P_{mn}} \frac{\partial^2 C}{\partial s_j^2}$$

This must be non-negative since $\frac{\partial P_{mn}}{\partial \gamma}$ negative implies that an increase in required revenue to be raised through the fee structure could be accomplished by lowering at least some of the prices. But this violates the original utility maximization as discussed earlier.

Case 1. Efficiency considerations only.

If we again assume existence of an optimal income distribution in the sense of Theorem 1, then (12) appears

$$\frac{\partial P_{mn}}{\partial \gamma} = \delta m \frac{\partial C}{\partial s_j} \Big/ \left[1 - \delta_m \gamma \sum_t \frac{\partial s_{tm}}{\partial P_{mn}} \frac{\partial^2 C}{\partial s_j^2} \right]$$

Thus, if the cost function can be approximated by a linear function in two periods[11] $\frac{\partial P_{mn}}{\partial \gamma} = \delta_m \frac{\partial C}{\partial s_j}$, such that if the pool is operating at different levels in the two periods, the optimal rate of change of the periodic fees will be different.

Case 2. Efficiency and Equity Considerations Necessitated.

In this case the full form of equation (12) must be evaluated and compared among periods: (a) If we begin from a uniform fee structure, then changes in γ which yield different rates of change in the fees will tend to differentiate the fee structure; (b) On the other hand, beginning from a differentiated fee structure, changes in γ may yield a tendency toward uniformity; or (c) To further divergence. At best, it appears that uniformity is an "unstable" (with respect to the revenue constraint) equilibrium. This is illustrated in Figure 1.

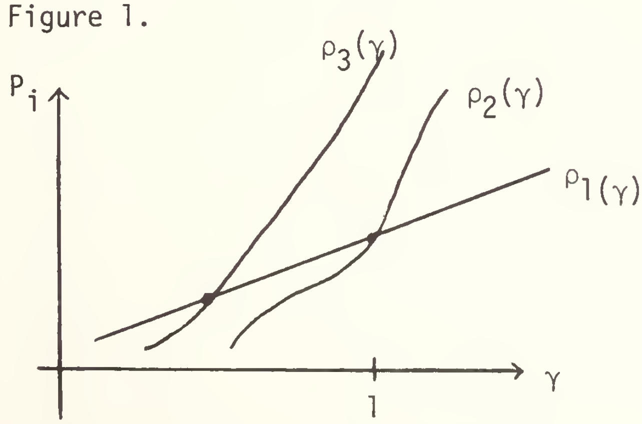

Figure 1

[11]Recall that we are not speaking of operation at capacity in this case and therefore the linear approximation is not unrealistic.

The combination of functions ρ_1 and ρ_2 illustrate situation (a) above, while that of ρ_1 and ρ_3 illustrate (b) and that of ρ_2 and ρ_3, (c). Only if all periodic rates-of-change- functions for a given age group are coincident with respect to γ will a uniform fee structure be optimal and stable. This, as inspection of (12) and (13) indicate, is not likely.

IV. IMPLICATIONS FOR DATA REQUIREMENTS.

We want to turn now to a discussion of the meaning and implications of the foregoing results for real-world pricing of publicly provided goods and services.

Again using our swimming pool as the point of discussion, suppose that the systems analyst consulting the director of parks and recreation elicits the decision that the "objective" of operating the pool is to maximize the use of the pool, while ensuring that its revenues cover operating costs. In this case the optimization problem becomes fairly simple in that the conditions the solution must satisfy include those which require that the admission price set for each period must be that at which net marginal revenue from admissions equals either zero or the shadow price of the capacity constraint, whichever is larger. Thus, given the existence of a large portion of "fixed" operating costs if the pool is open at all, if it can be established that there exist different demands for swimming in the various periods as a function of price, then the optimal fee structure will <u>not</u> be uniform. This possibility could be established through the collection of either historical or experimental data on observed attendance in, for example, the morning <u>vs</u> evening (or afternoon) with a given uniform price. A continuing experiment in varying the uniform prices could lead to an estimation of the form of the separate periodic demand functions, which would then be used to set the estimated globally optimum (differentiated) fee structure for the pool.

Of course, if the objective function is not quite as simplistic--say it is rather to maximize the attendance of a target group such as we discussed earlier--then the data collection problem becomes somewhat more complex. If the constraint remains as above--that is, to meet a revenue minimum--then an experiment such as discussed earlier <u>may</u> be sufficient* to establish

*Depending on the percentage of the target group who presently use the pool, and the extent to which this can be validated.

estimates of the general populace demands for swimming, but additional experimental market or survey data will be needed. Thus, a survey of a low income area which is identical with the residence space of the target group could be used to establish present use at prevailing prices and one or two estimates of use at possible alternative prices, in the various time periods. This would be used to estimate the set of periodic target group demand functions which could then be compared and contrasted with those of the general populace. If they proved to be statistically significantly different, then they would be used in the newly structured optimization problem to drive the set of estimated optimal periodic prices. These would very likely approach the "free swim in the off-peak-significant entrance fee in the peak period" type of pricing sometimes used at present, but not widely.

Finally, consider the problem in the case where, although a target group may be identifiable, it is not possible either to separately estimate their demand functions for the publicly provided service or to exactly specify the likely impact on overall utility of changes in the pricing policy. We submit that the analysis presented in the previous section lends credence to the position that a non-uniform pricing structure should be adopted at the time of the opening of the facility if it can be reasonably expected that total demand for the use of the facility will vary with time, ceteris paribus. This type of information could be fairly easily generated through a general populace telephone survey prior to the opening. The result--i.e. the initially set differential fee structure--would not necessarily be "optimal," but it would in all probability yield a higher level of community utility from the pool than a uniform fee. In addition, a gradient approach to selective marginal changes in the fees in the individual time periods may lead at least to a local optimum.

V. CONCLUSION.

In the context of the swimming pool problem setting, our results indicate that much more thought need be given to setting different peak and off-peak admission fees within a given age group. For example, given that the pool was built for a lower income segment of the community, and that their demand functions for swimming in a given period differ from those of the rest of the community *with respect to price*, then lower prices or even "free swims" in early afternoon or morning periods and correspondingly higher admission fees in the peak late afternoon/evening periods may yield greater total utility to the target group and keep the non-target group at least as well off as *before* the pool was built, while still satisfying the revenue constraint.

Even if the pool is a neighborhood facility (where there is only *one* social group), the model suggests that uniform fee structure may not be optimal if demands can be characterized as peak and off-peak.

In the general case, there seem to be many public or quasi-public facilities which by differentiating their fee structure may yield superior use patterns and hence utility gains. The full application of a model such as this may be difficult given the size and composition of the groups which are charged with setting such fees. However, approximations are possible and, we believe, desirable using fairly straightforward data gathering experiments.

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